

Econ 301: Managerial Economics

Topic 5A: Rational Choice Theory

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What is Rationality?

- A term that is thrown around a lot is ‘Rationality’.
- The foundation of economics is the idea of ‘Rational Choice Theory’ and thus rationality.
- But what is **Rationality**?
 - Spend 2 minutes writing down what you think ‘rationality’ and more importantly what ‘irrationality’ means!
 - What does it mean if someone is acting rationally
 - Given an example
 - What does it mean if someone is acting irrationally?
 - Given an example

What is Rationality?

- What is **Rationality**?
- In Economics, Rationality is simply that a person's preference satisfies 2 things:
 1. They are **Complete**
 2. They are **Transitive**.

Preferences

- As you know, economics is about making choices.
 - We want a framework where we can compare the choices that people make.
- **Revealed Preference:** A person who chooses x over y , must prefer x to y .
- How do we model these preferences?
- Formally speaking, a preference is a '**relation**'.

Preferences

- Here are some examples of relations:
 1. Alf is older than Betsy
 2. France is bigger than Luxembourg
 3. Sue is the mother of Franklin
 4. John is worried he may not do as well in this course as Jennifer

Each of these sentences expresses some sort of **binary relation** between **two** entities.

- Bruce stands between Clark and Diana is not binary (ternary relation).

Preferences

We can rewrite “Alf is older than Betsy”

As aRb , where:

a = Alf (the first entity)

R = the binary relation, in this case “is older than”

b = Betsy (the second entity)

Preferences

- Who/What are we comparing? Sometimes, this matters! (do we want to compare dogs ages to humans).
 - When we want to be careful, we can define a universe **U**, as the set of all things that can be related.
- E.G: Suppose the Universe is Ed, Edd, and Eddy. We write the universe U with all relevant items, separated by commas.
 - **U**= {Ed, Edd, Eddy}
- Order does not matter, so it could also be written as
 - **U**= {Eddy, Ed, Edd}
- Note: Universes can be infinite
 - The set of real numbers.
 - Points of time between 7:00 AM and 8:00 PM.
 - Points of time between 7:59 AM and 8:00 PM.

Preferences

- When it comes to preferences, a relation we care about is “at least as good as.” The symbol we use for these relations is $\succsim$
- This is known as the weak preference relation.
 - The “weak” wording here is important. You will see shortly why we say “at least as good as” and not “better than”.
- Examples:
 - For Ben, Coffee is at least as good as Tea.
 - $C \succsim_{\text{Ben}} T$ or $C \succsim_B T$
 - For Adam, Tea is at least as good as Coffee
 - $T \succsim_{\text{Adam}} C$ or $T \succsim_A \text{Coffee}$

Preferences

- So, what makes preferences rational?
- We only need two properties!
 - This simplicity is what makes the model so appealing

1. Completeness: Either $x \succsim y$ or $y \succsim x$ or both (for all x, y)

2. Transitivity: if $x \succsim y$ and $y \succsim z$, then $x \succsim z$ (for all x, y, z).

A rational preference relation is a preference that is both complete and transitive.

Transitivity

Transitivity: if $x \succcurlyeq y$ and $y \succcurlyeq z$, then $x \succcurlyeq z$ (for all x, y, z).

If the Universe is all people currently alive:

Is the binary relation of “is taller than” transitive?

Transitivity

Transitivity: if $x \succcurlyeq y$ and $y \succcurlyeq z$, then $x \succcurlyeq z$ (for all x, y, z).

If the Universe is all people currently alive:

Is the binary relation of “is taller than” transitive?

Yes!

- If Broden is taller than Zach
- And Zach is taller than Mark
- Then Broden is taller than Mark

$B \succ Z, Z \succ M$, therefore $B \succ M$.

Transitivity

Transitivity: if $x \succcurlyeq y$ and $y \succcurlyeq z$, then $x \succcurlyeq z$ (for all x, y, z).

- If the Universe is all people currently alive:
- What about the binary relation of “is in love with”?

Transitivity

Transitivity: if $x \succcurlyeq y$ and $y \succcurlyeq z$, then $x \succcurlyeq z$ (for all x, y, z).

If the Universe is all people currently alive:

What about the binary relation of “is in love with”?

No (fortunately)!

- If Steve “is in love with” Nancy And Nancy “is in love with” Jonathan”.
- Transitivity would mean Steve “is in love with” Jonathan”!

(While this may be true in some cases for 3 people in the universe), showing that transitivity is violated once is sufficient to prove the binary relation “is in love with” is not transitive.

Transitivity

Other examples:

Transitive:

- Is siblings with
 - Ron is siblings with Ginny, and Ginny is siblings with Fred $\rightarrow$ Ron $\succ$ Fred
- Is an ancestor of
 - Archie Manning is an ancestor of Cooper Manning, Cooper is an ancestor of Arch, Archie $\succ$ Arch
- Runs faster than
 - Usain Bolt runs faster than Prof Grodeck, Prof Grodeck runs faster than "student in this class", Bolt $\succ$ Student in this class

Transitivity

Not Transitive:

- Is a parent of
 - Archie Manning is the parent of Cooper Manning, Cooper is the parent of Arch, Archie is the parent of Arch **x**
- Is friends with
 - Person A friends with B, B friends with C, A is friends with C **x**
 - You don't have to like your friend's friend!
- Defeated (in the NFL)
 - Ravens defeated the Bears, Bears defeated the Steelers, Ravens defeated the Steelers **x**
- **Note:** A relation not being transitive, is not necessarily a bad thing! It just has that property. However, as you will see it is a problem for rational choice theory.

Transitivity

- Why does transitivity matter?
 - Without it, preferences can become unstable and leads to paradoxes (money pumps).
 - Necessary for meaningful utility representation of preferences.
- $U = \{\text{apple, orange, banana}\}$
- If you prefer **apples** to **oranges**
- and **oranges** to **bananas**,
- With transitivity, you prefer **apples** to **bananas**.
- BUT! Without the axiom of transitivity, you could prefer **bananas** to **apples**?
 - Why is this bad?

Money Pump

- This creates a loop where
 - apples $>$ oranges,
 - oranges $>$ bananas
 - bananas $>$ apples.
- The Money Pump: Intransitive preferences can be exploited (paradoxes in choice).
 - Z= Oranges
 - Y= Apples
 - X= Bananas

Completeness

Completeness: Either $x \succcurlyeq y$ or $y \succcurlyeq x$ or both (for all x, y)

- For any x and y in a universe, either x relates to y , y relates to x , or both.
- Is 'taller than' a complete relation?

Completeness

Is 'taller than' complete?

- No!

Proof: Chris Paul and Steph Curry are both 6'2 (1.88 m).

We cannot say:

- Chris Paul is taller than Steph Curry
- Steph Curry is taller than Chris Paul
- And we cannot say both

Completeness

This is why **weak** preference relations are important $\succsim$.

- There is an important link between **indifference** and the weak preference relation.
- I am indifferent between x and y ($x \sim y$) if and only if I weakly prefer x to y ($x \succsim y$) and I weakly prefer y to x ($y \succsim x$).
- Therefore, the weak preference relation includes the possibility of indifference.

A quick tangent: Logic

- I am indifferent between x and y ($x \sim y$) if and only if I weakly prefer x to y ($x \succsim y$) and I weakly prefer y to x ($y \succsim x$).
- What does “if and only if mean”?
- In Logic, if and only if (or iff) means that something is both a **necessary** & a **sufficient** condition (Logic form: $x \leftrightarrow y$)
- **Necessary condition:** y cannot happen without x : (Logic Form: $y \rightarrow x$)
 - **Example:** Air is a necessary condition for human life.
- **Sufficient condition:** if x then y . (Logic Form: $x \rightarrow y$)
 - **Example:** Owning a Labrador is **sufficient** for owning a dog.
- Note: They are logical counterparts (if x is a necessary condition of y , then y is a sufficient condition of x).

A quick tangent: Logic

1. **Necessary** Conditions:

- Having fuel is **necessary** for a car engine to run.
- Having a valid password is **necessary** to log into an account.
- Having oxygen is **necessary** for human survival

2. **Sufficient** Conditions

- Being divisible by 4 is **sufficient** for being even.
- Owning a Labrador is **sufficient** for owning a dog.
- Scoring 100% on an exam is **sufficient** to pass the exam

3. **If and only if** (Necessary and sufficient)

- An integer is even **if and only if** it is divisible by 2.
- Someone is a bachelor **if and only if** they are an unmarried adult male.

Completeness

- Going back to our definition of ‘indifference’, which is vital to preference orderings:
- I am indifferent between x and y ($x \sim y$) **if and only if** I weakly prefer x to y ($x \succcurlyeq y$) and I weakly prefer y to x ($y \succcurlyeq x$).
- Completeness definition: For all x and y , either $x \succcurlyeq y$ or $y \succcurlyeq x$ (or both).
- “or both”: This is just how we logically define indifference.

Completeness

Is 'at least as tall as' complete?

- **Yes!**

Example 1: Chris Paul and Steph Curry (both 6'2)

- Chris Paul is at least as tall as Steph Curry
- Steph Curry is at least as tall as Chris Paul
- In this case, we can say both!

Example 2: Victor Wembanyama (7'4) and Lionel Messi (5'7)

- Wemby is at least as tall as Messi

Example 3: Socrates (??) and Julius Cesar (??). Even if you don't know what two people's heights are, we can still say that Either:

- Socrates was at least as tall as Cesar
- Cesar was at least as tall as Socrates
- Or both!

Completeness

Some more examples:

1. Siblings with?
2. At least as good as (Universe= All basketball players)?
3. At least as good as (Universe= All people)?
4. At the same university as?

Note:

- To prove a relation is incomplete, you only need to show 1 example in the Universe where it does not hold.
- Proving completeness is a bit more difficult and beyond this course.

Completeness

Some more examples:

1. Siblings with?
 - **No!** It is possible that neither $x \succcurlyeq y$ nor $y \succcurlyeq x$
 - A mistake people make is to assume the opposite relation (either is siblings or is not siblings).
2. At least as good as (Universe= All basketball players)?
 - **Technically yes!** but good luck getting agreement on $\text{Lebron} \succcurlyeq \text{Jordan}$ or $\text{Jordan} \succcurlyeq \text{Lebron}$ (or both).
3. At least as good as (Universe= All people)?
 - More context needed! At least as good as at what? Can we compare Messi to Lebron? Ronaldo to Magnus Carlson? Tom Brady to Sabrina Carpenter?
4. At the same university as?
 - **No!** My friend Alex is at Uchicago.
 - Ben is at the same university as Alex is not true
 - Alex is at the same university as Ben is not true.
 - Both are not true

Completeness: Indifference

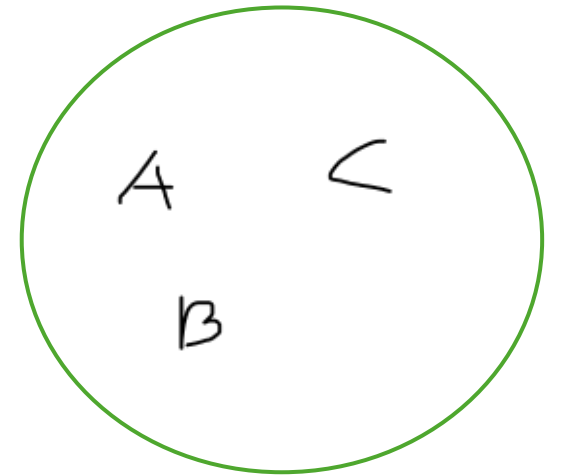
- **Note:** Indifference ($X \sim Y$) is not the same as incompleteness.
- With indifference you could flip a coin and be happy with either outcome
- Similarly, if I offered you \$1 along with either option, you would select the option with the \$1

Completeness

- Why do we care?
- If we want to be able to make inferences about people's preferences this means that for every 2 options in the Universe (set of options), a person must either:
 - Prefer one of the two options
 - or be indifferent between them
- Ben must either
 - Prefer apples to oranges
 - Prefer oranges to apples
 - Or be indifferent (see them as equally good).
- Ben cannot say “I simply cannot compare them”. The saying “apples and oranges” does not apply. We must be able to compare everything!

Preference Orderings

- When preferences are rational, we can create a ranking of all options from most to least preferred.
 - We can construct a preference ordering. A simple ranking of what a person prefers
- **Completeness** guarantees that there will be only one ordering. There are no gaps or ambiguous comparisons
 - If you have three options (A, B and C), you must be able to rank them fully, such as A is better than B is better than C, or I am indifferent between A and B which are both better than C.
- **Transitivity** guarantees there will be no cycles of strict preferences.
 - No money pumps



What counts as rational?

- Bob always chooses the option that results in him being paid less. Is this rational?
- According to this definition of rationality....**yes!**
- This definition of rationality, accordance to the axioms of completeness and transitivity, **provides little constraint on preferences**, while allowing for diverse preferences and behaviours.
- Weird preferences can still be considered rational
 - Preferring less money to more
 - Preferring money amounts that end in 3, to any other amount
 - More for someone else than yourself (Social Preferences)
 - Getting hit in a leg with a crowbar

Linking Rationality to Utility

- **Completeness** and **Transitivity** form the Foundation for Rational choice theory.
- However, there are others that can be added to provide more structure to preferences.
- We will now add some extra components

Do you want to play a game?

I will flip a coin:

- If it's Heads I will give you \$5.
- If it's Tails you will give me \$1

Do you want to play a game?

I will flip a coin:

- If it's heads I will give you \$5.
- If it's tails you will give me \$1

How do we value/calculate this gamble opportunity?

In other words, if you accept this gamble, what is the **Expected Value** of your decision?

Expected Value

Expected Value: the mean or average of a random outcome. We calculate this by taking a weighting average of the outcomes.

Expected Value (EV) Formula:

$$EV = \sum_{i=1}^n p_i \cdot x_i$$

Where:

- p_i = the probability of outcome i
- x_i = the value (or payoff) of outcome i
- n = the total number of possible outcomes

Expected Value

Coin Flip Example:

- n=2 outcomes
- \$5 with 50% chance
- -\$1 with 50% chance

$$EV = \sum x_i * p_i$$

$$EV = (5 * 0.5) + (-1 * 0.5)$$

$$EV = \$2$$

Expected Value

Another Example: If you rolled a die, and I paid you the number you rolled*\$1. What is the **Expected Value** of rolling the die?



Expected Value

N= 6 (possible outcomes). Probability (p_i) of these outcomes added together =1

$$EV = \sum x_i * p_i$$

EV= The sum of all the outcomes*probability of that outcome occurring

$$EV = \$1*(1/6) + \$2*(1/6) + \$3*(1/6) + \$4*(1/6) + \$5*(1/6) + \$6*(1/6) = \$3.5$$

Note: The EV is simply the mean!

- EV=What we expect to get on average (in the long run).



What would you rather?

What would you rather?

1. Receive \$330 for certain
2. Receive \$100 multiplied by the number shown on a rolled die?

Expected Value

1. \$330
2. \$350

- Does that mean preferring 1 over 2 is irrational?

How much is this gamble worth

- Consider the following game. I will toss a coin until it lands on tails. If **tails** happen on the **n'th toss**, you get $\$2^n$ dollars. How much would you pay to play this game?
- How much would you pay to accept this gamble?
 - What is the fair price?
 - What is the Expected Value of this gamble?

How much is this worth: The St Petersburg Paradox

How many possible outcomes?

Outcome 1: T= \$2

- Pr=0.5

Outcome 2: HT= \$4

- Pr=0.25

Outcome 3: HHT= \$8

- Pr=0.125

Outcome 4: HHHT= \$16

- Pr=0.0625

Outcome n:.....

- The Expected Value of this gamble is infinity!
 - Why do people pay less than infinity? Rarely willing to pay more than \$10 to \$20!

Utility from Money

People don't maximize Money. They maximize Utility.

Expected Value does not account for:

1. Risk Preferences
2. Diminishing Marginal Utility of Money

Risk Preferences

Would you rather:

Gamble 1: \$1,000 for certain

Gamble 2: \$0 with 60% chance or \$3,000 with 40% chance? $EV = \$1,200$

$EV(2) > EV(1)$

- It's not irrational to prefer Gamble 1 to Gamble 2 despite the EV of Gamble 2 being higher. Appetite for risk matters!

When does \$500 not equal \$500

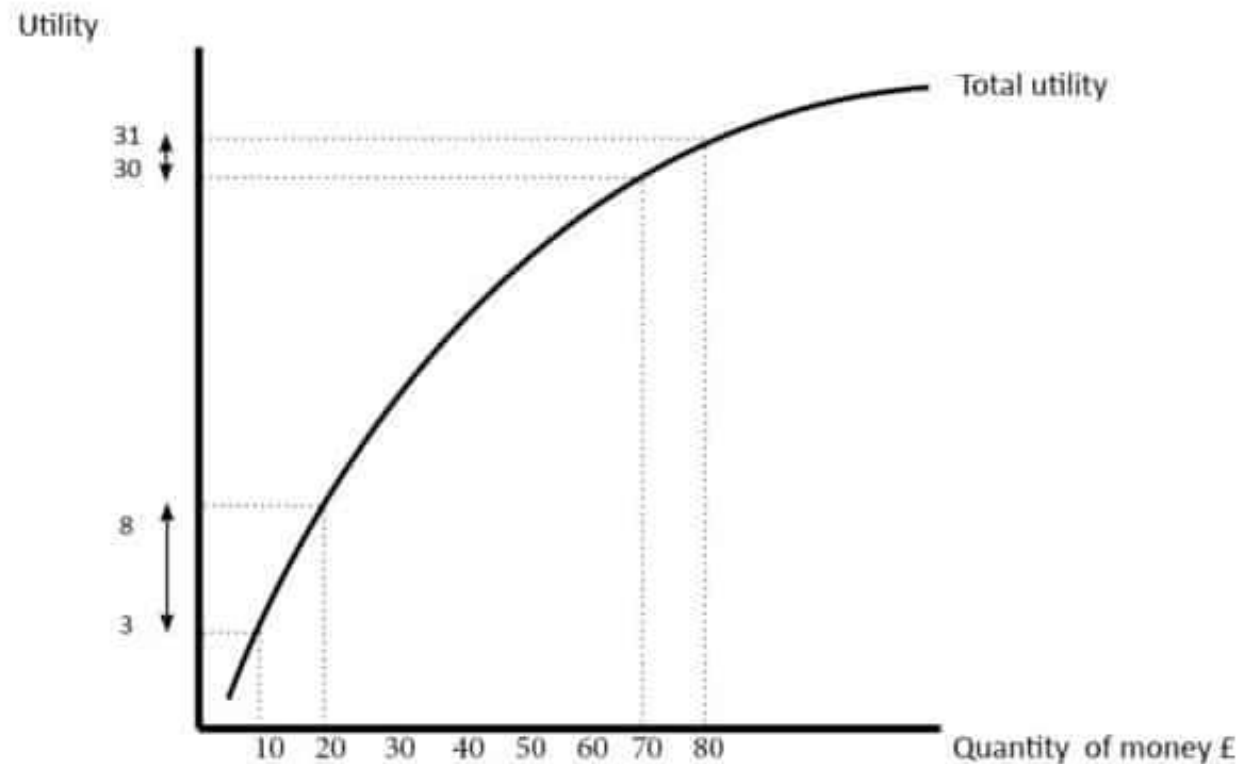
- Is \$500 always \$500?

Who would value \$500 more?

- Person A: Has \$0 in their bank account
- Person B: Has \$1 million in their bank account
- Current wealth matters!

Diminishing Marginal Utility of Money

- Individuals do not make decisions based purely on wealth, but rather on the utility of their wealth.

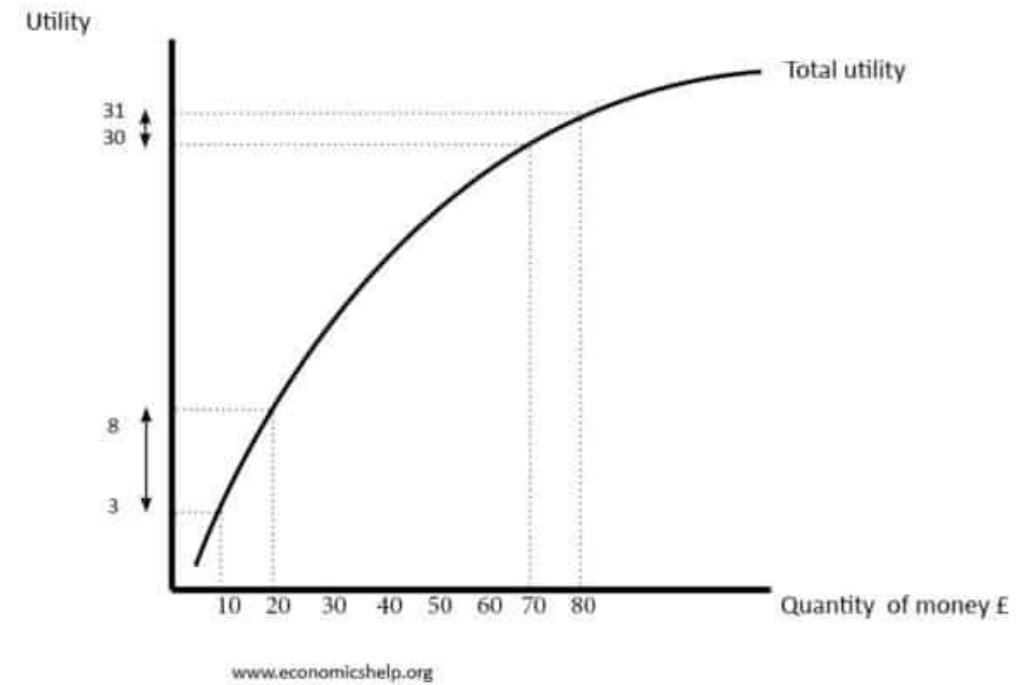


Diminishing Marginal Utility of Money

- If Marginal Utility is decreasing then “ $EU < EV$ ”
- Diminishing Marginal Utility for ‘most goods’.
 - You get more pleasure from eating your first burger than the 5th burger
 - A team that has won 0 SuperBowls would arguably get more pleasure from winning the SB in 2025/26 than a team that won last year.
- What matters is the utility of the prizes (in total).
 - E.g., if a person’s utility function $u(x) = \ln(x)$
 - A person with \$1000 should pay about \$11 for the St Petersburg gamble

Diminishing Marginal Utility of Money

- Utility functions are $u=x^y$, where $0<y<1$
 - E.G: $X^{0.5}$
- Utility is always increasing in money
 - $f'(x)>0$
 - $\frac{du}{dx}=0.5X^{-0.5}$
- The amount it increases by is decreasing, all becoming less each time (second derivative is negative)
 - $f''(x)<0$
 - $\frac{d^2u}{dx^2}=-0.25X^{-1.5}$



Expected Utility Theory

Expected Utility: The sum of the utility of each outcome multiplied by the probability of that outcome occurring.

Expected Value: $EV(x) = \sum x_i * p_i$

Expected Utility: $EU(x) = \sum u(x_i) * p_i$

EU can take many forms. For Example:

- $u(x) = \ln(x)$
 - $u(x) = x^{0.5}$
 - $u(x,y) = x^\alpha y^{1-\alpha}$ for all $\alpha = [0,1]$
 - Classic Microeconomics choice problem
-
- This means we use **Utility Functions**. The utility function is a (or the!) key construct in economic theory.

Utility Functions: What are they?

- The assumption of rational choice allows us to use a utility function to represent the preferences of individuals.
 - Preferences must be **Complete** and **Transitive**
 - In other words: Utility maximization is equivalent to maximization of a **complete, transitive** preference $\succsim$.
- **Utility theory**
 - We can use a utility function to represent the preferences of individuals. A utility function $u(x)$ assigns a numerical value to each element in X .

$$x \succsim y \Leftrightarrow u(x) \geq u(y)$$

$$x \succ y \Leftrightarrow u(x) > u(y)$$

$$x \sim y \Leftrightarrow u(x) = u(y)$$

Utility Functions: What are they?

- A utility function is just a scale to **rank** different alternatives. The numbers themselves don't have any *cardinal* meaning.
 - A utility function **can** tell us that a person prefers apples to oranges.
 - A utility function **can not** tell us that a person prefers apples by 73 utils to oranges.

Utility Functions: CAN

For example: $u(\text{banana})=1$; $u(\text{apple})=2$

- $u(a) > u(b) \Rightarrow a \succ b$

Definition of Utility: A utility function $u(\cdot)$ can map the set of choice alternatives into the set of real numbers and can represent a preference relation if the following is true:

- $u(a) \geq u(b)$ if and only if $a \succcurlyeq b$ (for all a and b).

Utility Functions: CANNOT

Example: if $u(\text{banana})=1$; $u(\text{pear})=3$

- We **cannot** say that pears are preferred 3x more than bananas.
 - Utility is an ordinal ranking.
 - Assigns larger numbers to preferred alternatives
 - The magnitude of the numbers do not matter.
- $u(\text{pear})=3 > 1 = u(\text{banana}) \Leftrightarrow \text{pear} > \text{banana}$

Expected Utility Theory

- In 1947, von Neumann and Morgenstern proved that preferences can be represented by a utility function if they satisfy the following axioms:
- Completeness
- Transitivity
- Continuity (beyond this course)
- Independence
 - Independence of Irrelevant Alternatives

Expected Utility: $EU(x) = \sum u(x) * p$

Independence

Independence: If $x > y$, then for any z , $[p, x; (1-p), z] > [p, y; (1-p), z]$

- $x+z \geq y+z$ iff $x \geq y$ (i.e. a choice between two alternatives should depend only on how those outcomes that differ)

Independence of Irrelevant Alternatives

IIRA: A new option shouldn't flip how you rank two existing options.

If $x \succcurlyeq y$ in choice set $S = \{x, y\}$ then:

If $x \succcurlyeq y$ in choice set $T = \{x, y, z\}$

Important because:

- Choices are context independent
 - Avoids manipulation
 - Simplifies economic models
-
- Distinct from Independence Axiom, which is only relevant for lotteries.

Invariance

- Not an explicit axiom, but would also be a violation of EUT.

Invariance: A Decision maker should not be affected by the way alternatives are presented.

- The ranking of preferences should not depend on the description of the options (*description invariance*) or on the method of elicitation (*procedure invariance*).

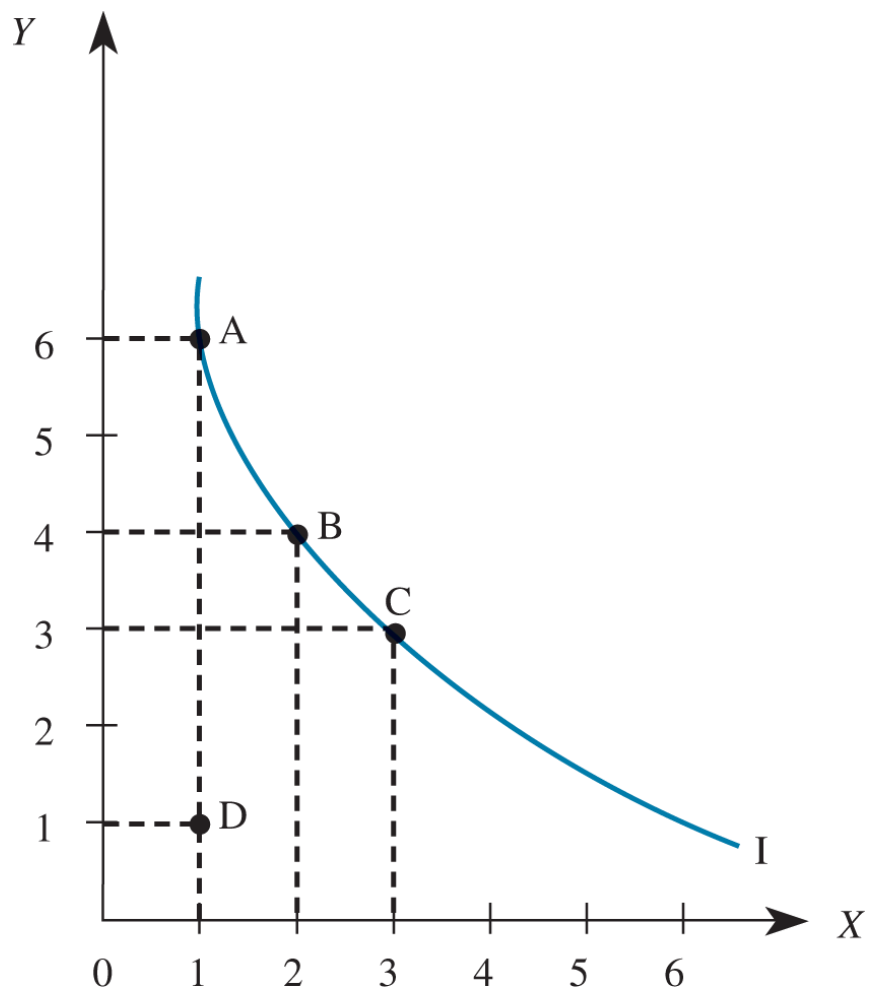
Invariance

Example 2: If you prefer a Danish to a doorstop, then you should not switch your preference based on a change in description.

Auxiliary Axioms of EUT

- These are not required for expected utility theory, but make analysis more practicable.
 - Important to construct **Indifference Curves!**
- **Utility is calculated over total wealth**
- **Non-satiation:**
 - You can never have enough utility. No upward limit of happiness
- **Monotonicity**
 - More of a good thing is always better
- **Convexity**
 - To be discussed
- **Diminishing Marginal Utility**

The Indifference Curves



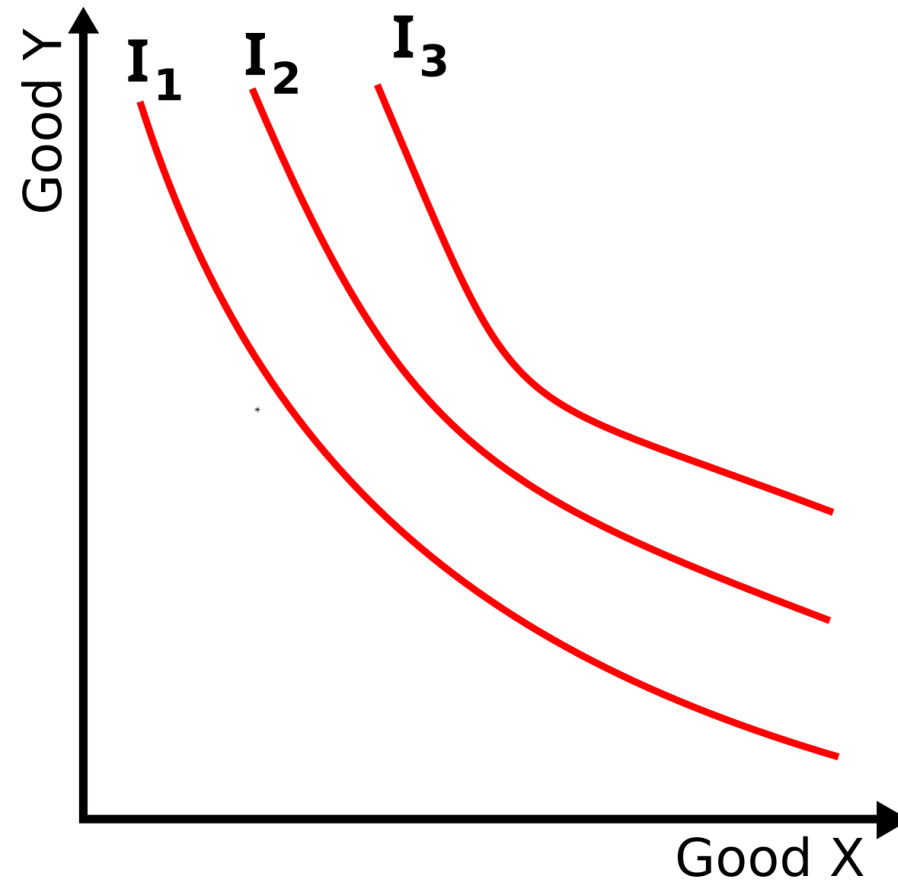
Indifference curve -

A curve that defines the combinations of two goods that give a consumer the same level of satisfaction

Why Indifference Curves

- We have discussed preferences for single items (apple v orange) or single attributes (speed v height etc).
- How do we think about preferences when there are different combinations of multiple goods (bundles)
 - Goods: (apples & oranges)
 - Attributes: Finding a partner (personality, looks)
 - Attributes: Choosing a college (campus life, academic rigor)
- Different combinations (of the same goods) mean there are many points of indifference.
 - 7 apples and 1 orange ~ 2 apples and 3 oranges
 - 9 campus life and 3 academic rigor ~ 6 campus life and 4 academic rigor
- How can we represent these preferences?

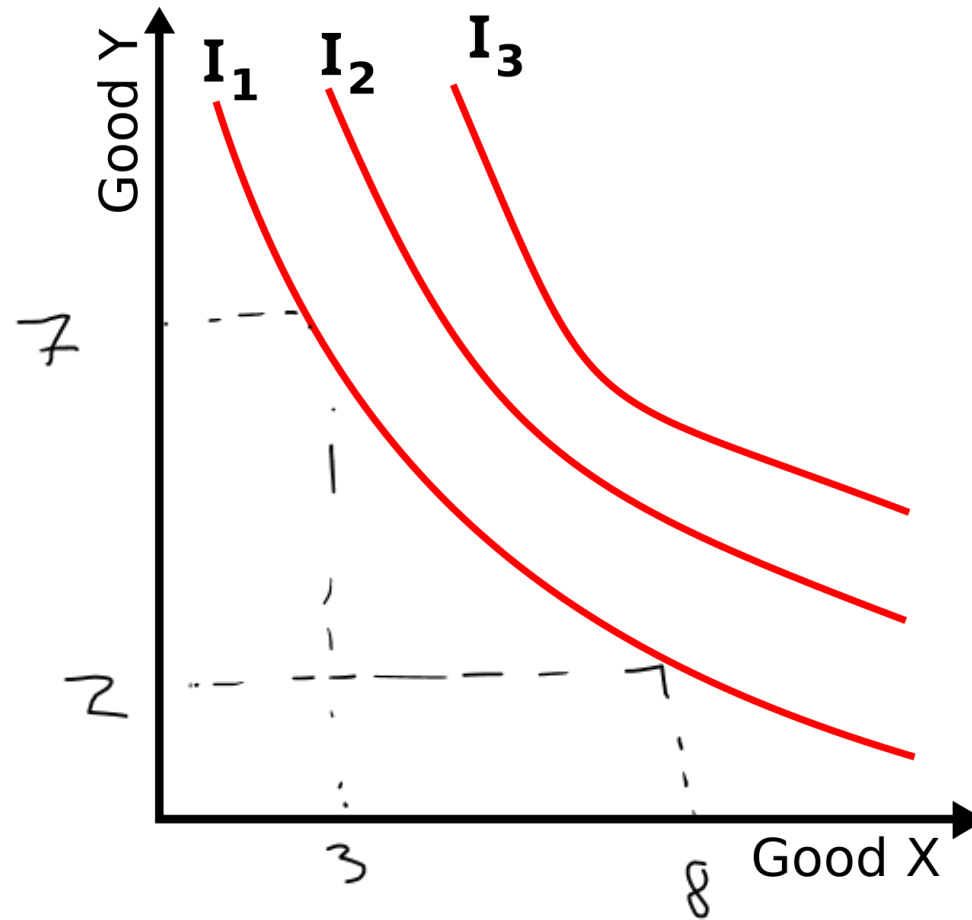
Indifference Curves



Indifference Curves Principles

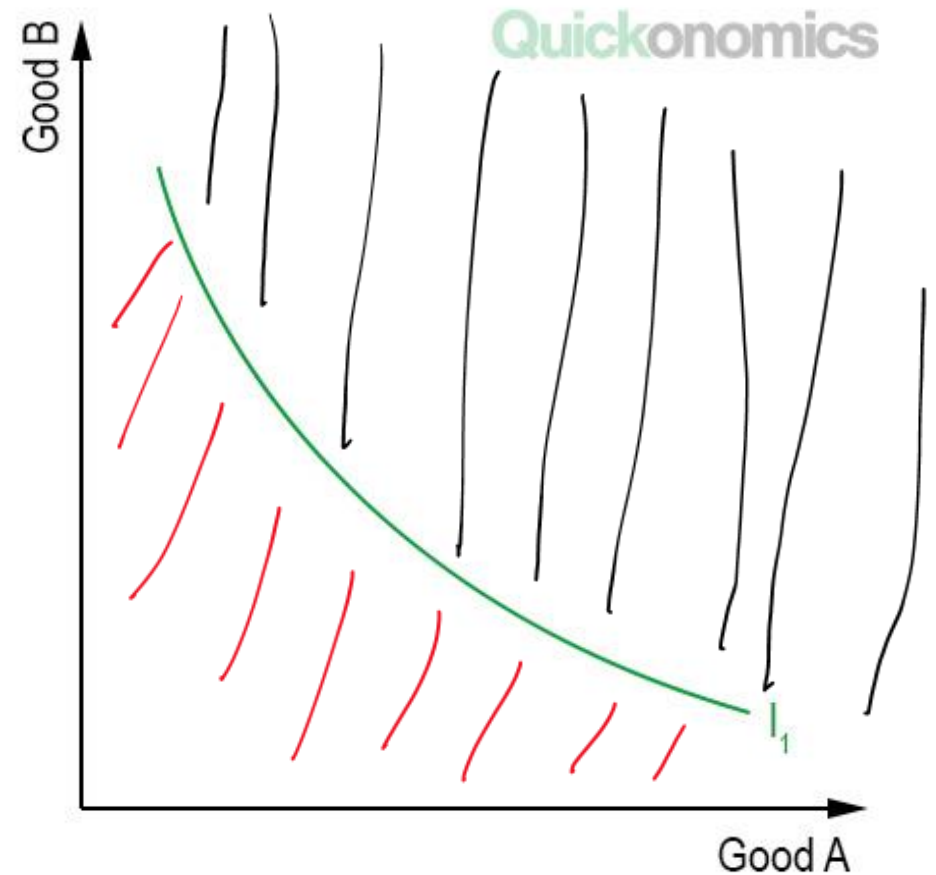
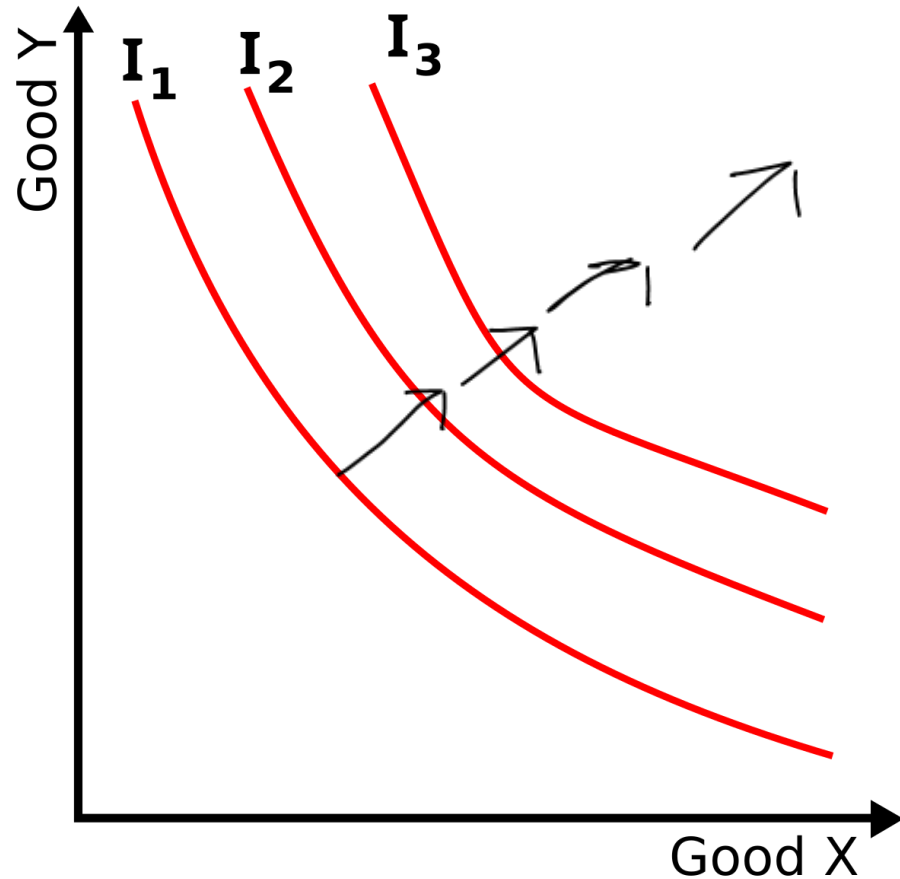
1. Indifference along a curve
2. Higher curve means higher utility
3. Cannot intersect
4. Convexity + Downward Sloping

1. Indifference along a curve

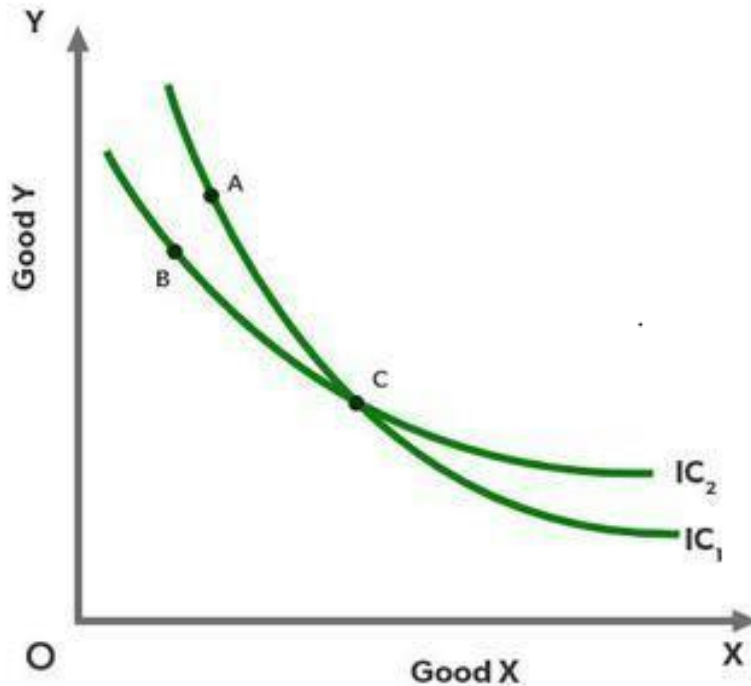


Indifferent between
the bundle of (3,7) and
(8,2).

2. Higher Curve Means Higher Utility



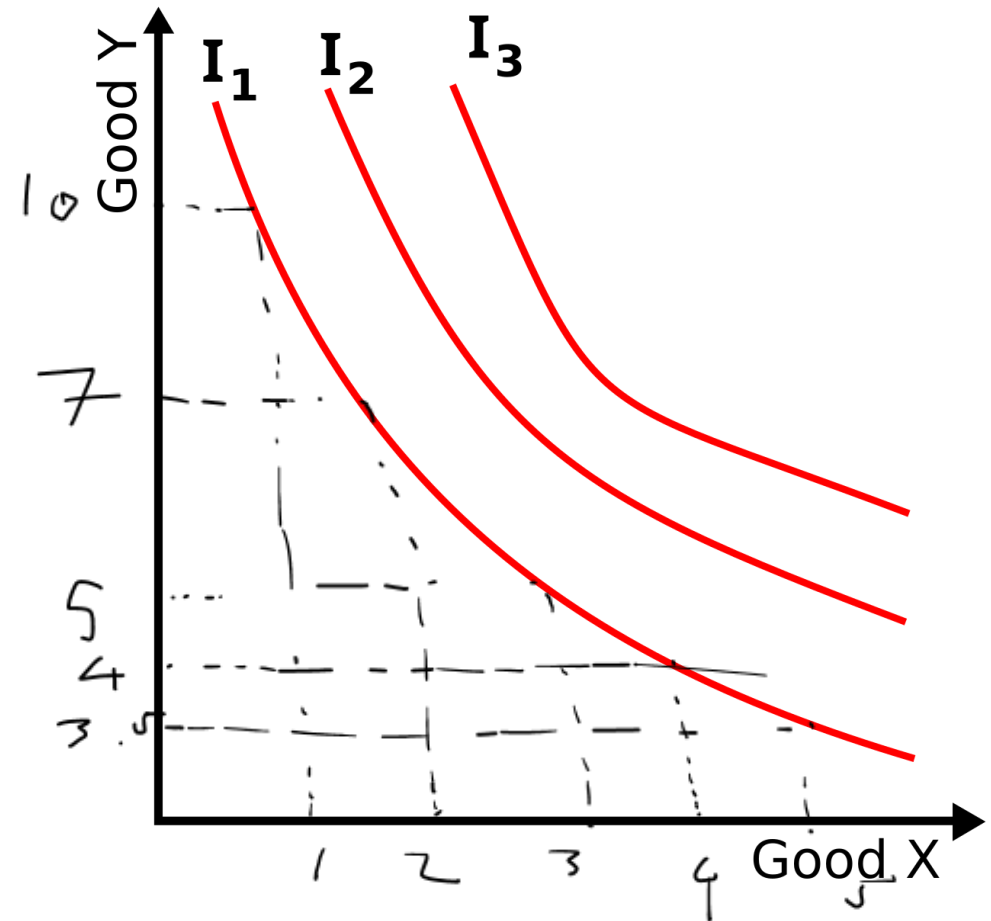
3. Indifference Curves cannot intersect



- Why can't they intersect?
- Violates transitivity
- Proof by contradiction:
- $A \sim C$, $C \sim B$, therefore $A \sim B$
- However, we know from "Higher Curve means Higher Utility" B is below IC_1 where A lies, so $A > B$
- $A > B$ and $A \sim B$ cannot both be true
 - Also note $A_x > B_x$ & $A_y > B_y$

4. Convexity + Downward Sloping

- **Downward Sloping:** This ensures that on a singular IC to gain more of one good, you have to give up some of the other.
- Diminishing marginal rate of substitution
 - The MRS changes as you move along a curve.
 - Originally you are willing to give up more Y to gain one extra X, but this diminishes as you gain more X.



Marginal Rate of Substitution

Example 1:

- Two Alternatives, both equally preferred (same value of utility and would be on same utility curve)
 - A= 6 hours of sleep, 70 on an exam
 - B=5 hours of sleep, 80 on an exam
- The **marginal rate of substitution** is someone's willingness to trade off one thing for another. This student is willing to trade sleep for points

$$MRS \text{ sleep for points} = \frac{|10|}{|-1|} = 10$$

Marginal Rate of Substitution

Example 2: Adding a third option

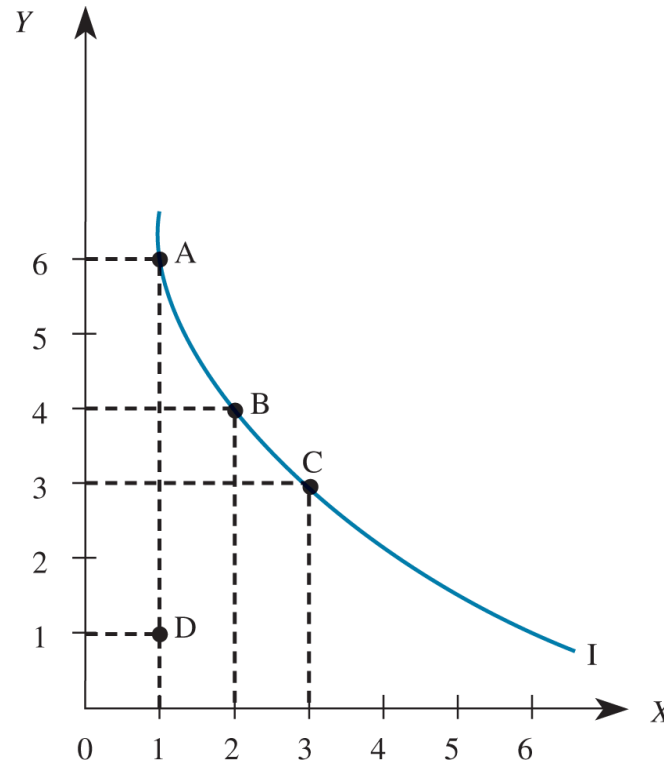
- A= 6 hours of sleep, 70 on an exam
- B=5 hours of sleep, 80 on an exam
- C=4 hours of sleep, 85 on an exam
- Going from B->C

$$MRS \text{ sleep for points} = \frac{|5|}{|-1|} = 5$$

- All make the student equally happy. So, at 5 hours of sleep, a student is willing to trade off 5 more points for 1 hour less sleep.

Marginal Rate of Substitution

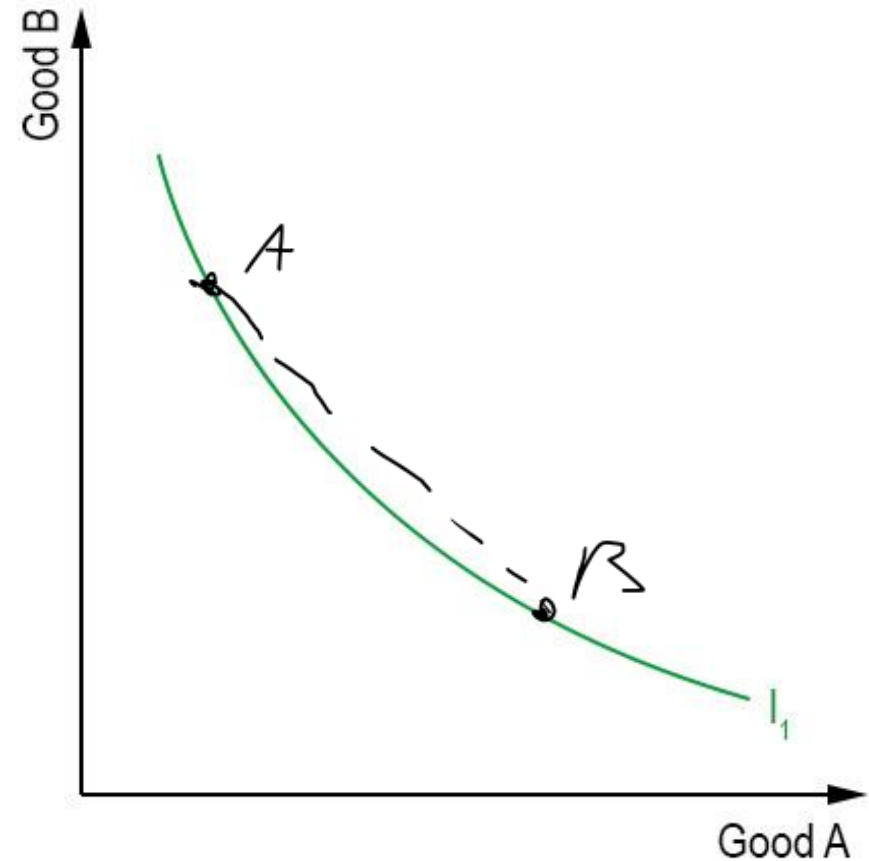
- In our indifference curves, this is just $MRS = \frac{MU(x)}{MU(y)}$



4. Convexity + Downward Sloping

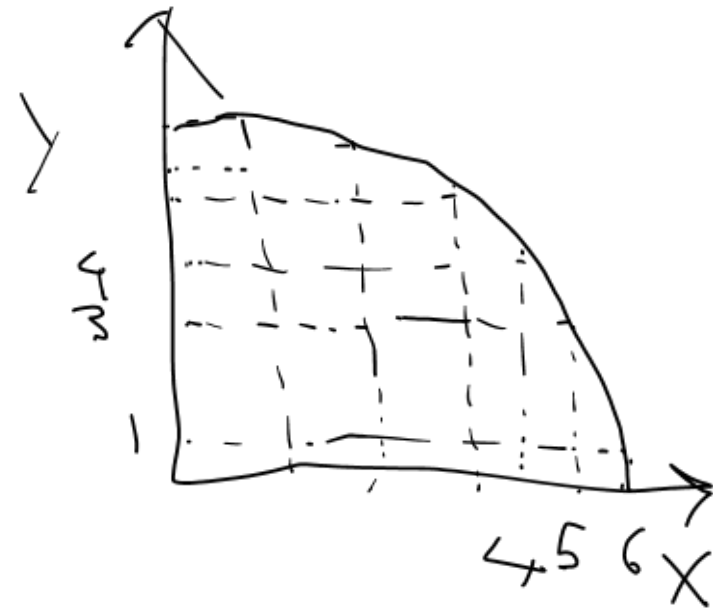
Convexity: When considering two bundles between which you are indifferent, you would prefer having any convex combination of those two bundles to either of the bundles themselves.

- A convex combination is just a fancy way of saying “weighted average.”
- Visually, preferences are convex if, for any two points on the same indifference curve, a segment connecting those two points passes through the “preferred region”
- **Note:** It is the preferences that are convex, not the utility function. Remember ICs map preferences only!

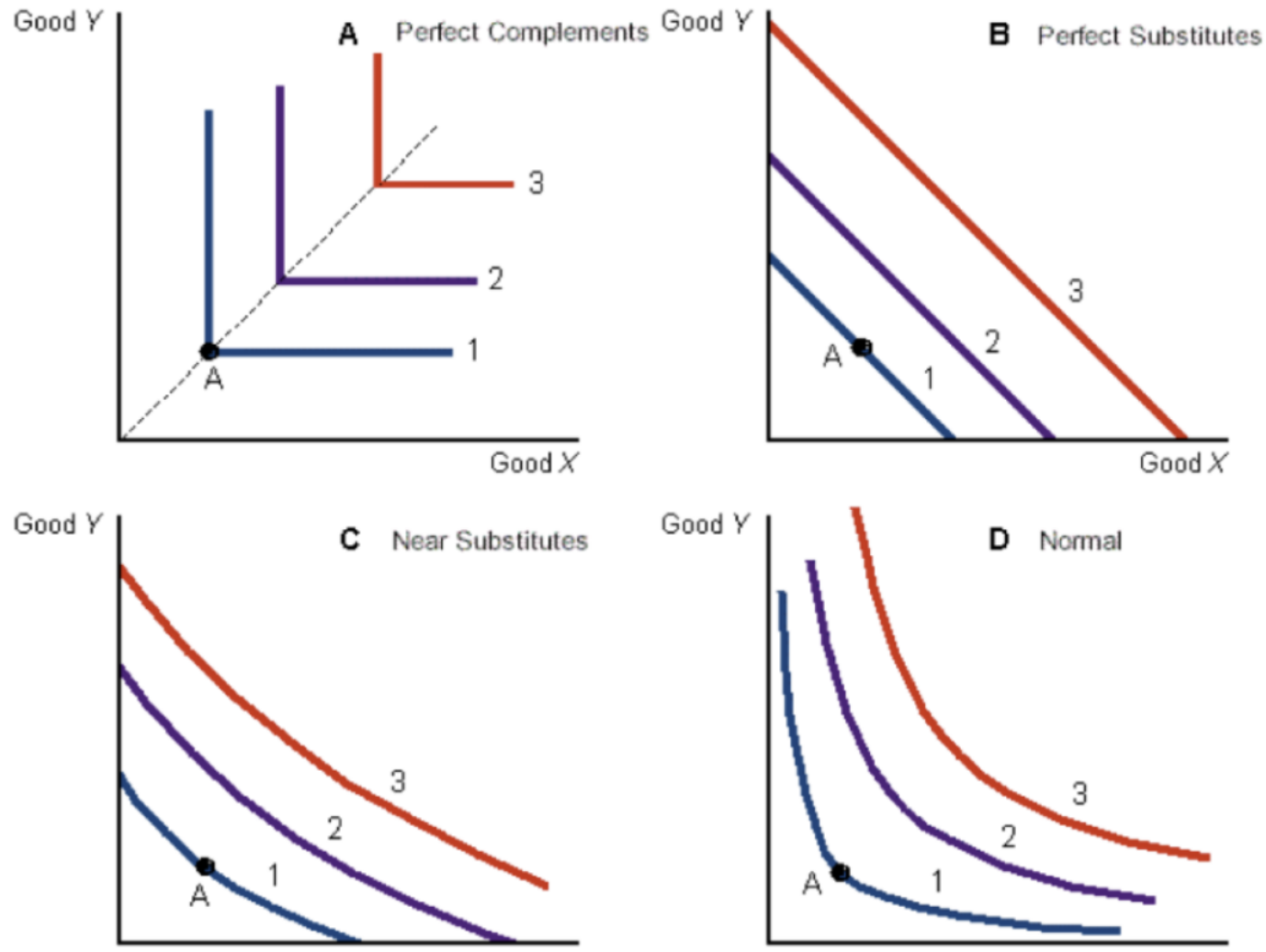


4. Convexity + Downward Sloping

- **Why don't we have concave curves?**
- A concave IC would imply that as you consume more of one good, you'd be willing to give up more of the other good for an additional unit of the first.
- This violates the principle that the satisfaction from additional units of a good declines (diminishing marginal utility).



Indifference Curves: Special Cases



Linking Indifference Curves to Utility Functions

Remember: We discussed the link between preferences (individual things, or bundles of things) and utility.

Utility theory: We can use a utility function to represent the preferences of individuals. A utility function $u(x)$ assigns a numerical value to each element in X .

$$a \succcurlyeq b \Leftrightarrow u(a) \geq u(b)$$

Linking Indifference Curves to Utility Functions

